AER1216: Fundamentals of UAVs
PERFORMANCE

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Introduction

What we will cover:

1. Simplified Standard Atmosphere
2. Simplified Airspeed measurements
3. Propulsion Basics
4. Fixed Wing Performance
5. Quadrotor Hover Performance

These set of Notes will only cover a very specific subset of UAVs

1. low-speed ($M < 0.2$)
2. low-altitude ($h < 500$m)
3. propeller driven (fixed pitch mainly)
4. Piston-engine or Brushless electric motors

Simplified Standard Atmosphere

Aircraft performance is strongly influenced by the air density, ρ;

$q = \frac{1}{2}\rho V^2$

Piston engine performance Power $= f(\rho)$

Thrust $= f(\rho)$

Simplified Standard Atmosphere

The general *Standard Atmosphere* is quite complicated and includes the variation of the gravity with height above the earth as well as the average temperature variation with respect to height through the earth's atmosphere. As we are only considering much lower altitudes than regular aircraft we will assume a constant gravity.
Assumptions:

- assume constant gravity, $g = g_{SL}$
- use the hydrostatic equation $dp = -\rho g dh$
- temperature varies $T = T_{SL} + ah$ where $a = -6.5C^{\circ}/1000$m.
- ideal equation of state holds $p = \rho RT$

Starting with equations of state

$$dp = -\rho g dh$$

using the equation of state, re-arranging, and integrating we get,

$$\int \frac{dp}{p} = \int \frac{g}{RT} dh$$

plugging in our expression for temperature variation and evaluation the integral from SL to height H yields,

$$p_H = p_{SL} \left(\frac{T_{SL} + ah}{T_{SL}} \right)^{\frac{-g_{SL}}{aR}}$$

and

$$\rho_H = \rho_{SL} \left(\frac{T_{SL} + ah}{T_{SL}} \right)^{-\left(\frac{g_{SL}}{aR} + 1 \right)}$$

Note: these equations look *Exactly* like the full standard atmosphere equations, the only difference is h here is the regular altitude rather than the geo-potential altitude.

Simplified Standard Atmosphere

The values to use in these equations are as follows:

$$p_{SL} = 101325 \text{ Pa}$$

$$\rho_{SL} = 1.225 \text{ kg/m}^2$$

$$T_{SL} = 15\text{C}^\circ \text{ or } 288.15\text{K}^\circ$$

$$g_{SL} = 9.80665 \text{ m/s}^2$$

At 500m the following quantities are obtained using our model

$$p = 95461 \text{ Pa}$$

$$\rho = 1.1673 \text{ kg/m}^2$$

Of course the real atmosphere does not equal the standard atmosphere and the sea-level values provided above can be adjusted as required. The point to note is that the density decreases with altitude as does the pressure which well effect the UAV performance

Airspeed

Of particular concern to UAVs is the speed relative to the air. As wind speeds can be substantial, the speed of the UAV relative to the earth as measured by GPS will not tell us the entire story for our UAV.
In fact, pressure measurements are usually employed. The velocity of airflow is obtained by measuring the difference between the total and static pressure.

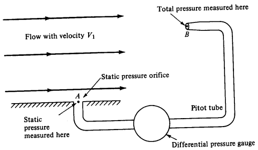
- static pressure is the pressure that would be measured when moving with the flow,
- total pressure is the pressure that results when the flow is isentropically slowed to zero speed

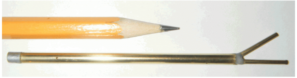
Airspeed

Consider incompressible flow, Bernoulli's equation is,

$$p + \frac{1}{2}\rho V^2 = \text{constant} = p_t$$

- *Static pressure* p is the local atmospheric pressure
- *Dynamic pressure* $q = \frac{1}{2}\rho V^2$ represents the pressure associated with the flow motion
- *Total pressure* p_t is also measured by a pitot probe in which the moving air mass is brought to rest





Airspeed

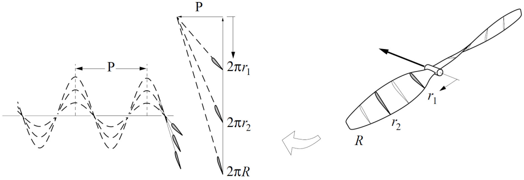
A pressure sensor can measure the difference between total and static pressure ($p_t - p$), thus using incompressible Bernoulli equation

$$V = \sqrt{\frac{2\left(p_t - p\right)}{\rho}}$$

we can calculate the airspeed. Of course we don't usually know the local density. Sea-level density is used to get the indicated airspeed, V_{IND}. For low-speeds this also yields the EQUivalent airspeed which is the airspeed that would lead to the same dynamic pressure at standard sea-level conditions. A static pressure sensor can be used to calculate standard altitude and then a temperature sensor can correct to get true airspeed using the equation of state. Alternately weather charts can be used to correct for density. Note dynamic pressure, $q = 1/2\rho V^2$ is important by itself and we have measured this with our pitot tube for low-speeds.

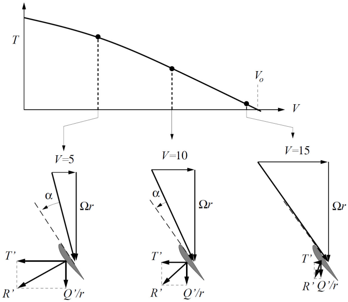
Note we have neglected an important part of airspeed measurements. As the aircraft itself modifies the pressure distribution it can be difficult to get an accurate measure for the free-stream static pressure. Corrections are usually made for this based on careful flight test measurements, these corrections convert from indicated airspeed to calibrated airspeed. This is beyond the scope of this lecture however.

A propeller is generally defined by its diameter, D, and its pitch, P. The chord distribution is also important for propeller design, but for small aircraft the selection of the range of chord distributions is limited.



The pitch of a propeller is defined in the same way as a woodscrew: it is the distance that the propeller would travel in one revolution if advanced through a solid material. A 9 $\times$ 4 propeller has a diameter of 9 *in* and a pitch of 4 *in*.

For a given propeller the Thrust, T is a function of forward speed and RPM of the propeller. For a fixed RPM as the airspeed approaches the pitch speed (V_p) the angle of attack on the blades, and thus the thrust, approaches 0. Note that often propeller blades are cambered so that we need to define α with respect to the zero-lift-line to have the lift $=0$ at $\alpha=0$.



Increasing either the motor RPM, or the pitch of the propeller will increase V_p and the maximum speed of the aircraft.

Propeller behaviour can be succinctly captured in non-dimensional charts of C_T, C_P and η versus J [1], where:

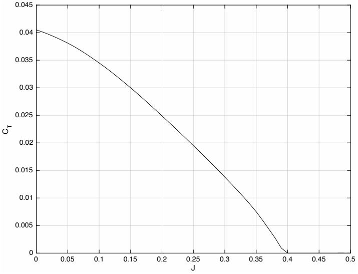
$$J = \frac{V}{nD} \quad C_T = \frac{T}{\rho n^2 D^4} \quad C_P = \frac{P}{\rho n^3 D^5} \quad \eta = \frac{TV}{Q\Omega} = \frac{C_T J}{C_P}$$

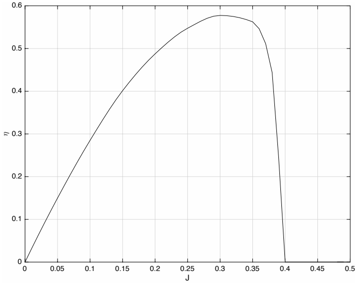
where n =rev/sec=$\frac{\Omega}{2\pi}$, P=Input Power=Q Ω, and Q=torque. Ideally these charts are from measurements but often they can also be approximated using numerical techniques such as modified blade element theory.

[1] Often there may still be slight changes in C_T and C_P with rpm

Propulsion-Propellers

Here are some hypothetical charts for a 10x5 propeller[2]:





For our 10inch propeller this gives a pitch speed of 0.1016 m/s per rev/s, so at 8000RPM this works out to 13.5 m/s.

[2] These charts are for a propeller with nothing but the motor behind it–the fuselage of the aircraft will further lower the thrust and efficiency

Propulsion-Electric Motors

Depending on the application of the UAV, for small UAVs electric motors provide a good alternative to internal combustion, piston engines. They are quieter, easier to start, more reliable and cleaner. For long-duration flights however the power density in fuel may still favour internal combustion engines for UAVs.

We will only consider Brushless DC motors here, they are very efficient, light electric motors that are usually powered by LiPo batteries and require an Electronic Speed Controller (ESC), that provides both the commutation and change in voltage (using PWM) for these motors.

The performance of a typical brushless electric motor can be approximated with the following set of equations:

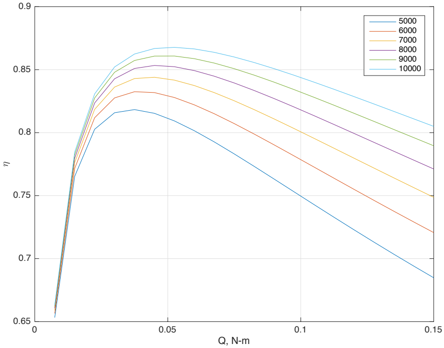
$$\begin{aligned} \Omega &= K_v(\upsilon - ir) \\ Q &= K_t(i - i_0) \\ Q\Omega &= (\upsilon - ir)(i - i_0) \end{aligned}$$

K_v characterizes how the back EMF of the motor, $\upsilon - ir$, varies with RPM. The back EMF is the voltage generated across the motor's terminals that opposes the drive voltage as the windings move through the motor's magnetic field. r is the internal resistance of the armature, and i_0 is the no-load current at the specified motor voltage.
The equations equates the shaft power to the electric power minus electric losses, and upon substituting the torque Q and angular rate Ω into the equation for shaft power we get,

$$K_v K_t = 1 \;\; \frac{\text{rad/s}}{\text{V}} \frac{\text{Nm}}{\text{A}}$$

Propulsion-Electric Motors

Using these equations, the motor current, RPM and efficiency can be plotted against the terminal voltage and torque. The AXI Goldline 2217/16 motor ($K_v = 1050 RPM/V$, $i_0 = 0.4A$, and $r = 0.12 ohm$) is shown below where RPM and torque are treated as the independent parameters.

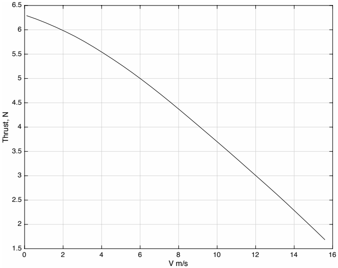


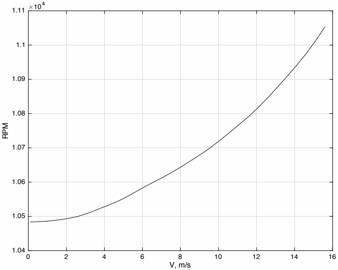
These motor characteristics can then be combined with the propeller charts to get the thrust, current, rpm, and power versus speed curves for the given motor/propeller. The solution is found by iterating as follows:

1. For a range of airspeeds, V and a given voltage v
2. Assume rotational speed, Ω
3. Calculate J using Ω and V and D
4. Get Thrust from C_T versus J chart, convert to T.
5. Get η from η versus J chart and using $TV = \eta Q \Omega$ get torque, Q
6. From motor equations get current, i
7. From motor equations get new rotational speed, Ω_n
8. if $|\Omega_n - \Omega| > \epsilon$ go to 3

Propulsion-Electric Motors

For our Goldline AXI 2217/16 motor, with $v = 11.1$ volts (3 cell LiPo) and our hypothetical 10x5 propeller this gives the following plots,



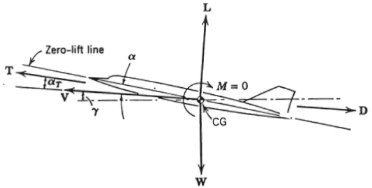


Total efficiency can also be calculated once the current and voltage into the motor are known and the thrust and speed are known. One further factor needs to be considered however which is the efficiency of the ESC which is usually between 80 % and 95%. Careful selection of the propeller, motor, battery, ESC is required to maximize the efficiency for the cruise speed of the aircraft.

Propulsion-Gas Motors

Internal combustion, piston engines are also commonly used in UAVs. The behaviour of these motors are very complicated and depend on a large number of factors. Larger four-stroke engines steady state behaviour can often be captured in charts that related power output and fuel consumption at full-throttle to rpm and manifold pressure. For our simplified analysis if the propeller is well matched with the motor and flight condition we can consider engine as producing constant power for a given air density and the fuel consumption is proportional to the power produced.

Equations of Co-Planar Flight



The four forces acting on the airplane:

1. Lift, which is perpendicular to the flight path direction
2. Drag, which is parallel to the flight path direction
3. Weight, which acts vertically toward the center of the earth
4. Thrust, which in general is inclined at the angle α_T with respect to the flight path direction

When the earth is considered flat, any earth-fixed frame of reference is an inertial system. We will derive the desired differential equations. It is most convenient to express the forces in components $\perp$ and $\parallel$ to $\vec{V}$.

$$T\cos\alpha_T - D - W\sin\gamma = m\dot{V} \quad (1)$$

$$T\sin\alpha_T + L - W\cos\gamma = mV\dot{\gamma} \quad (2)$$

Note that if α_T is very small, then $\sin\alpha_T \approx 0, \cos\alpha_T \approx 1$. That leads to

$$T - D - W\sin\gamma = m\dot{V} \quad (3)$$

$$L - W\cos\gamma = mV\dot{\gamma} \quad (4)$$

For level ($\gamma = 0$), unaccelerated flight (right side of above equations are zero),

$$T = D \quad (5)$$

$$L = W \quad (6)$$

Thrust Required, Level, Steady flight

Given the drag polar of the aircraft is given by,

$$C_D = \underbrace{C_{D_0}}_{\text{parasite drag or zero lift drag}} + \underbrace{\frac{C_L^2}{\pi e AR}}_{\text{induced drag}} \tag{7}$$

The required thrust, T_R, to fly in this condition is[3],

$$T_R = \underbrace{q_\infty S C_{D_0}}_{\text{zero lift} T_R} + \underbrace{q_\infty S \frac{C_L^2}{\pi e AR}}_{\text{induced} T_R} \tag{8}$$

and since for level unaccelerated flight

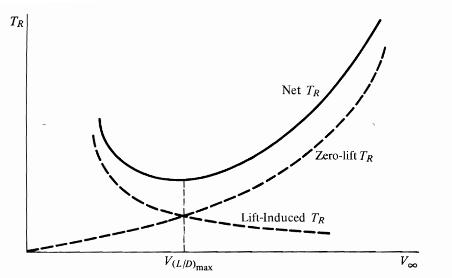
$$C_L = \frac{W}{q_\infty S} \tag{9}$$

[3] The $()_\infty$ indicates that the dynamic pressure or velocity is the free-stream condition away from the aircraft

Thrust Required, Level, Steady flight

$$T_R = q_\infty S C_{D_0} + \frac{W^2}{q_\infty S \pi e AR} \quad (10)$$

Thus since $q_\infty = 1/2\rho_\infty V_\infty^2$ the zero lift T_R varies with V_∞^2 and the induced T_R varies with $1/V_\infty^2$



Since $C_L = C_{L_\alpha}\alpha$ and $L = W$, α increases as V_∞ decreases. From the above figure we can see that for some thrust settings we can fly slower with larger angle of attack or faster with smaller angle of attack.

Thrust Required, Level, Steady flight

Now multiply the two Equations 5 and 6 to get the thrust required, T_R, to fly in this condition,

$$T_R = \frac{W}{L/D} = \frac{W}{C_L/C_D} \tag{11}$$

and to minimize the required thrust we obviously need to maximize the C_L/C_D or simply the L/D.

The *lift-to-drag ratio* L/D is a measure of the aerodynamic efficiency of an aircraft. The maximum aerodynamic efficiency leads to the minimum thrust required.

Thrust Required, Level, Steady flight

$$E = \frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D_0} + KC_L^2} \tag{12}$$

The maximum E happens at $\frac{dE}{dC_L} = 0$ (C_{D_0} is fixed for given aircraft):

$$\frac{dE}{dC_L} = \frac{(C_{D_0} + KC_L^2) - C_L(2KC_L)}{(C_{D_0} + KC_L^2)^2} \tag{13}$$

$$= \frac{C_{D_0} - KC_L^2}{(C_{D_0} + KC_L^2)^2} = 0 \tag{14}$$

leading to the maximum condition

$$C_{D_0} = KC_L^2 = C_{D_i} \tag{15}$$

and

$$E_{max} = \frac{C_L}{C_D} = \frac{1}{2\sqrt{KC_{D_0}}} \tag{16}$$

and

$$V_{T_R min} = \sqrt{\frac{2W}{\rho_\infty S}\sqrt{K/C_{D_0}}} \tag{17}$$

Thrust-to-weight ratio

To keep level flight at a given altitude, the thrust required must have:

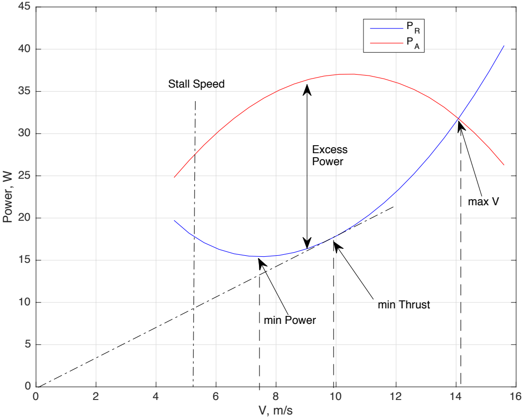
$$\frac{T}{W} \geq 2\sqrt{KC_{D_0}} = \frac{1}{E_{max}} \tag{18}$$

and the minimum thrust required happens when the lift-to-drag ratio is at its maximum

T_R is an airframe-associated phenomenon, while T_A (thrust available) is strictly associated with the propulsion unit. The intersection of the T_R curve with the T_A curve gives the maximum velocity of the aircraft (at a given altitude for level flight). However instead of plotting required and available thrust usually plots of required power (P_R) and available power (P_A)are used.

Using our same motor/propeller combination ($P_A = T_A V$, and $P_R = T_R V$) and for a small aircraft with wing area, $S = 0.5\text{m}^2$,C_{D_0}=0.04, W=20N, $e = 0.8$, $AR = 6$ ρ=1.225kg/m^2 we get the following plot,

Power Required



The minimum power condition, minimum thrust, and maximum speed can all be read off this one plot!

Power Required

The $P_{R_{min}}$ condition is particularly important as for a piston engine with well matched propeller the fuel consumption is proportional to the power. For an electric motor we have a fixed amount of energy in our battery and the weight of the battery is constant, so power will also determine how long we can fly on a single charge (assuming we match the propeller/motor well to the given speed).

Power is defined as energy per unit time. Therefore, the *power required* for a level, unaccelerated flight at a given altitude and a given velocity is

$$P_R = T_R V_\infty = \frac{W}{C_L/C_D}\sqrt{\frac{2W}{\rho_\infty S C_L}} \propto \frac{1}{C_L^{3/2}/C_D} \tag{19}$$

Similar to our analysis for minimum thrust, It can be shown that the minimum power required happens when

$$\frac{dP_R}{dV_\infty} = 0 \Rightarrow C_{D_0} = \frac{1}{3} C_{D_i} \tag{20}$$

The airspeed to fly at this condition is given as,

$$V_{P_R min} = \sqrt{\frac{2W}{\rho_\infty S}\sqrt{\frac{K}{3C_{D_0}}}} \tag{21}$$

Range-Piston Engines

The total distance travelled on a tank of fuel (or Range, R) depends on the complete flight which usually includes a climb segment, a cruise segment and a descent segment.

We will analyse a simpler situation: *what is the maximum distance an aircraft can fly, at cruising altitude and airspeed, on a given amount of fuel*.

To cover the longest distance, common sense says that we must use the minimum fuel consumption per unit distance (e.g. km or mile).

Then for steady, level flight, we have

$$\frac{-dW}{dt} = cP_{eng} \tag{22}$$

where W is the weight of the airplane (which only changes due to fuel burn) and the negative sign indicates loss and c is the constant between power and weight of fuel burnt. In other words the incremental reduction in weight of the airplane $-dW$ is due to fuel consumption over a time increment, dt. Since the distance travelled ds is equal to $V_\infty dt$, the incremental distance travelled by the plane is,

$$ds = V_\infty dt = \frac{-V_\infty}{cP_{eng}} dW \tag{23}$$

but $P_A = P_{eng}\eta$ and in steady level flight $P_A = P_R$ and $P_R = DV_\infty$ so this becomes,

$$ds = -\frac{V_\infty \eta}{cDV_\infty} dW = -\frac{\eta}{cD} dW \tag{24}$$

but $L = W$ and $C_L/C_D = L/D$ therefore integrating from $s = 0$ where $W = W_0$ to $s = R$ where $W = W_1$ this becomes,

$$R = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{C_L}{C_D} \frac{dW}{W} \tag{25}$$

and assuming constant c, η, C_L, and C_D this integrates to,

$$R = \frac{\eta}{c} \frac{C_L}{C_D} \ln \frac{W_0}{W_1} \tag{26}$$

This is the classic *Breguet range formula* which gives a quick practical range estimate that is accurate within about 15%.

Range-Piston Engines

For maximum range C_L/C_D and propeller efficiency needs to be maximized. Substituting in the expression for $(C_L/C_D)_{max}$ we found earlier leads to,

$$R_{max} = \frac{\eta}{2c}\sqrt{\frac{\pi eAR}{C_{D_0}}}\ln\frac{W_0}{W_1} \tag{27}$$

Notice R is independent of density (and hence altitude) for the propeller aircraft. To get this range you will need to fly at $V_{T_R min}$ so the maximum range will be covered in a shorter time at higher altitudes. Also the speed will need to be adjusted as fuel is used.

Range Electric Aircraft

Weight is constant as the battery is discharged. If we consider the total energy in the battery as E and the power required as $P_R = \eta T_R V$ then

$$\frac{dE}{dt} = \eta T_R V \tag{28}$$

and the distance is given as,

$$\frac{ds}{dt} = V \tag{29}$$

so

$$\frac{dE}{ds} = \frac{T_R}{\eta} \tag{30}$$

if we assume η is constant, [4]this is minimized by always flying at the minimum thrust condition, $T_{R_{min}}$ or maximum C_L/C_D, and the range is given as,

$$R = \frac{\eta E}{T_{R_{min}}} \tag{31}$$

This is achieved at the same speed as for the Gas Powered Aircraft ($V_{T_R min}$) but now the weight is constant so speed is constant for a given altitude

[4]If we pick our motor propeller combination for optimal efficiency at the corresponding speed, then η will be maximum at $T_{R_{min}}$

We may also be interested in maximizing the time in the air, T_E rather than the distance traveled. Starting with Equation 22

$$dt = \frac{-dW}{cP_{eng}} \tag{32}$$

and given that $P_{eng} = P_A/\eta$ and for level flight $P_A = P_R = DV_\infty$ this yields,

$$dt = \frac{\eta}{cDV_\infty} dW = \frac{\eta}{cV_\infty} \frac{C_L}{C_D} \frac{dW}{W} \tag{33}$$

but $V_\infty = \sqrt{2W/\rho_\infty S C_L}$ so this becomes,

$$dt = \frac{\eta}{c} \sqrt{\frac{\rho_\infty S}{2}} \frac{C_L^{3/2}}{C_D} \frac{dW}{W^{3/2}} \tag{34}$$

and now integrating between $t = 0$ where $W = W_0$ and $t = T_E$ where $W = W_1$, we obtain:

$$T_E = \int_{W_1}^{W_0} \frac{\eta}{c} \sqrt{\frac{\rho_\infty S}{2}} \frac{C_L^{3/2}}{C_D} \frac{dW}{W^{3/2}} \tag{35}$$

and assuming c, C_L and ρ_∞ are constant,

$$T_E = \frac{\eta}{c} \sqrt{2\rho_\infty S} \left(\frac{C_L^{3/2}}{C_D} \right) \left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}} \right) \tag{36}$$

In order to achieve maximum endurance for a propeller driven airplane, we want the following:

- fly at low altitude
- maximum weight loss $W_0 - W_1 = W_f$, where W_f is the fuel weight.
- maximum $C_L^{3/2}/C_D$, i.e. fly at the speed, $V_{P_R min}$, where this is maximized for a given aircraft.

For a given altitude, the maximum endurance is achieved when $C_L^{3/2}/C_D$ is maximized.

Endurance Electric Powered Aircraft

Since the rate of energy use is given by the power the maximum endurance for an electric aircraft with a fixed available battery power E is achieved when the aircraft is flown at the minimum power condition, $P_{R_{min}}$. The endurance is simply,

$$T_E = \frac{E}{\eta P_A} \tag{37}$$

and the speed is such that we are flying at maximum $C_L^{3/2}/C_D$. Note that this speed is less than the speed for maximum range! Also note that we considered η fixed or maximized at the same speed as $P_{R_{min}}$. So we select our motor and propeller for optimal efficiency at this speed for this analysis to be meaningful.

Additional Performance Analysis

Similar analysis can be used to calculate:

- Maximum rate of climb
- Maximum sustained turn-rate
- Maximum ceiling for Gas Powered UAVs
- Take-off performance

Unfortunately due to time constraints we will not cover these topics today.

Summary Fixed Wing

For maximum endurance we want to design an airplane that has a large $C_L^{3/2}/C_D$ whereas for maximum range we should design an airplane that has a large C_L/C_D. We should also design the propulsion system to have its highest efficiency at this operating point (i.e. airspeed)

DISCLAIMER: Please note that this section is still a work in progress as I haven't really worked much with quadrotor performance! There may be better ways of performing this analysis that lead to more concise or more straightforward results. I still think it is useful to present however, just so you can get a flavour for the important parameters for at least the hover performance of a quadrotor. I have only considered quadrotors, the analysis could easily be extended to generic mult-rotors.

Quadrotors are operated quite differently from fixed wing aircraft. Typical applications of quadrotors require long periods of mainly hovering flight. From a performance point of view the hovering endurance would seem to be an important metric and will be the main metric we consider today. One could also consider maximum range for delivery but this is beyond the scope of today's lecture. Consider a quadrotor in hover, then for steady state:

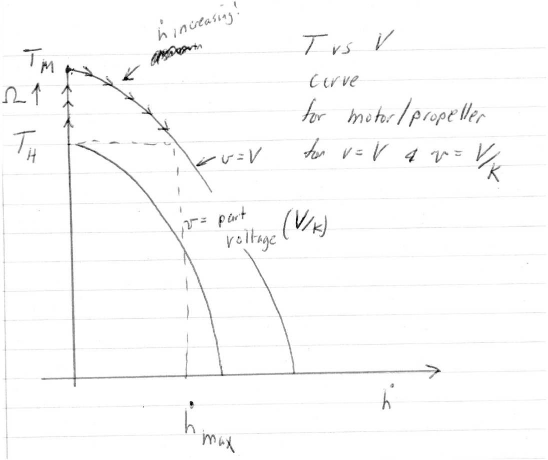
$$T_H = W/4 \tag{38}$$

Of course we need to be able to maneuver which will require thrust in excess of the hover value. If we consider the maximum thrust as

$$T_M = T_H\,(1 + A) \tag{39}$$

then the quadrotor would could climb with an maximum initial acceleration of A (in units of g's).

However as our vertical speed increases the thrust would decrease due to our propeller performance and would eventually settle it to a maximum rate as shown on the plot below:



The additional thrust can also be used to maneuver in other ways, such as rotating, pitching/rolling along with the additional horizontal speed that would accompany the pitch/roll motion. Rules of thumb from the industry suggest A should be 0.5 to 1.0 for good overall maneuverability. We would obviously like to hover efficiently which means we would like a propeller with a small C_P for a given C_T (at J=0 or the so called static thrust and power).

If we just consider the propeller efficiency to start with we can make some progress on picking an 'optimal' propeller to produce T_M and hence T_H efficiently. To have maximum hover time we would like to maximize,

$$E_{max} = \left(\frac{T_H}{P_{in}}\right)_{max} = \left(\frac{C_T}{C_P nD}\right)_{max} \quad \text{subject to } C_T \rho n^2 D^4 = T_H \qquad (40)$$

Quadrotor Performance

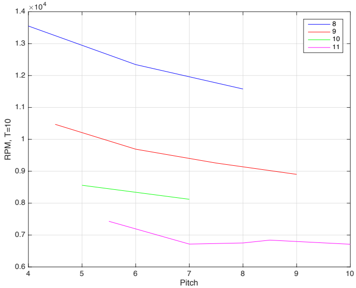
To date I have not found closed form expressions for determining the propeller for optimal hover but momentum theory tells us it is more efficient to use a larger actuator disc area with a smaller Δp across the actuator disc. For an example problem lets assume:

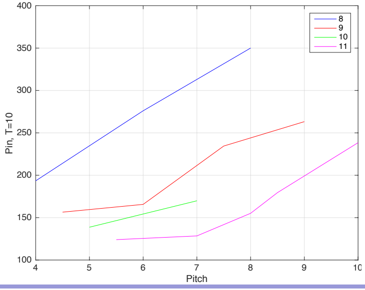
- $T_H = 10$ N
- We will assume that there are no Re effects due to changes in RPM. For real propellers the static C_T will decrease as the RPM is reduced due to Re effects. The Re effects on static C_P are less pronounced. We will use the C_T at an average RPM in this analysis.
- We will use a range of thin electric propellers from APC as performance charts are available from UIUC
- We will assume the same AXI 2217/16 electric motor in our previous analysis.

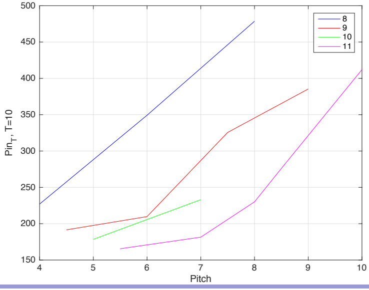
The analysis was performed as follows:

- for each propeller (with known D and ρ) use the static C_T to calculate the required RPM to generate 10N static thrust
- for each propeller calculate the required propeller input power using C_P, rpm, D,and ρ.
- for each propeller calculate the required torque by dividing power by the rotational speed (in rad/s)
- for the given RPM and torque Q determine the motor efficiency.
- divide the required propeller input power by the motor efficiency to find the total required power

Quadrotor Performance







These results suggest the following:

- Use a large propeller with a small pitch for optimal hover endurance. These results agree in general with simple momentum theory
- The advantages of the larger propeller start to diminish as the pitch goes towards zero. This would be even stronger if *Re* effects due to RPM were considered.

This analysis in quite simplistic however and other factors need to be considered, here are a few:

- the inertia of the propellers (which will increase quickly with D) must be considered. The motor will not be able to spin up the bigger propellers as quickly and this may effect maneuverability
- Very large propellers may lead to safety issues
- Very large propellers may lead to interference (aerodynamic or mechanical) between the the 4 propellers.